

Heat and Mass Transfer Study of Multiphase Flow of Viscous and Viscoelastic Fluids

Anuj Kumar Srivastav^{1*}, Neeraj Srivastava², Shaily Tyagi¹, Vimal Kumar¹

¹Bennett University, Greater Noida, UP, India

²Agra College, Agra, UP, India

012anuj@gmail.com

Abstract

As per this study, we investigate the flow behavior of two immiscible, electrically conducting fluids - one viscous fluid and one viscoelastic fluid - within a porous medium between two infinite horizontal plates. The research examines how these two fluids flow, given their different properties, in a confined porous space. As per the configuration, there are two fluid layers: the lower layer contains viscous fluid, and the upper layer contains viscoelastic fluid. As per the setup, these layered arrangements experience heat and mass transfer effects due to the various physical forces acting on them. The governing equations for flow, momentum, and energy transport are formulated as nonlinear coupled partial differential equations. This system itself describes the transport phenomena further through mathematical modeling. These equations are actually solved using mathematical methods with proper boundary conditions. The solutions definitely follow standard analytical approaches. We observe that the final expressions contain only coefficients, which we evaluate numerically for different parameter combinations. Also, basically, the coefficients are analyzed in detail using the same tabulated data format. Further, the changes in velocity and temperature profiles are shown in graphs to study how each fluid region itself responds to the physical parameters.

Keywords: *Viscous Fluid, Viscoelastic Fluid, Multiphase fluid, Mathematical modelling, Governing equations.*

1. Introduction

Basically, fluid dynamics studies how liquids and gases move, and the same principles explain many natural processes and technologies. In airplane and car making, we are seeing it helps reduce air resistance and increase lift through better wing and body shapes only, making vehicles use less fuel and stay more stable. Civil and hydraulic engineers actually use fluid dynamics to design pipelines, dams, canals, and drainage systems. This knowledge definitely helps them build structures that prevent floods and properly manage water flow. As per engineering requirements, understanding water flow and sediment movement is necessary for safe infrastructure design. This knowledge ensures proper and efficient construction work. Devakar and Raje (2019) numerically analyzed the unsteady MHD flow of immiscible Newtonian and micropolar fluids through a porous channel. Yadav et al. (2018) developed a mathematical model for micropolar fluid flow in a two-phase immiscible system through porous media. Kumar and Yadav (2023) explored the peristaltic motion and heat–mass transfer of MHD non-miscible Newtonian and micropolar fluids in a porous, asymmetric, saturated channel. Guo et al. (2021) have made extensive contributions to the diffuse-domain method for two-phase flows with a large density ratio in complex geometries. This study examines the heat and mass transfer behavior of viscous and viscoelastic

fluids in a porous channel confined

between two horizontal plates. The governing equations are solved using a combination of analytical and numerical methods, and the resulting velocity and temperature profiles are presented in graphs and tables to provide a detailed understanding of the flow dynamics.

2. Mathematical Formulation

For viscous fluid flow region

$$\frac{\partial \bar{u}_1}{\partial \bar{x}} = 0 \quad \dots (1)$$

$$\mu_1 \frac{\partial^2 \bar{u}_1}{\partial \bar{y}^2} - \bar{\mu}_1 \bar{u}_1 - \bar{\rho}_1 \bar{g} \beta_1 (T_1 - T_0) = \bar{q}_r$$

$$\bar{\mu}_1 \frac{\partial^2 \bar{u}_1}{\partial \bar{y}^2} - \bar{\mu}_1 \bar{u}_1 + \bar{\rho}_1 g \beta_1 (T_1 - T_0) = \bar{q}_r \quad \dots (2)$$

$$\bar{\rho}_1 c_p \frac{\partial \bar{T}_1}{\partial \bar{y}} - \bar{K}_1 \frac{\partial^2 \bar{T}_1}{\partial \bar{y}^2} - \bar{Q} = \bar{\rho}_1 c_p \frac{\partial \bar{T}_1}{\partial \bar{y}} - \bar{\rho}_1 c_p \frac{\partial \bar{T}_1}{\partial \bar{y}} = 0 \quad \dots (3)$$

For viscoelastic fluid flow region

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad \dots (4)$$

which gives $\bar{v} = -V_0$ (Constant)

$$\bar{\rho}_2 \frac{\partial \bar{u}_2}{\partial \bar{x}} + \rho_2 \bar{v} \frac{\partial \bar{u}_2}{\partial \bar{y}} = \bar{\mu}_2 \frac{\partial^2 \bar{u}_2}{\partial \bar{y}^2} - K \bar{v} \frac{\partial^3 \bar{u}_2}{\partial \bar{y}^3} - \bar{\mu}_2 \bar{u}_2 - \bar{\rho}_2 \bar{g} \beta_2 (T_2 - T_0) = \bar{q}_r$$

$$\bar{\rho}_2 c_p \frac{\partial \bar{T}_2}{\partial \bar{y}} - \bar{K}_2 \frac{\partial^2 \bar{T}_2}{\partial \bar{y}^2} - \bar{Q} = \bar{\rho}_2 c_p \frac{\partial \bar{T}_2}{\partial \bar{y}} - \bar{\rho}_2 c_p \frac{\partial \bar{T}_2}{\partial \bar{y}} = 0 \quad \dots (5)$$

$$\bar{\rho}_2 c_p \frac{\partial \bar{T}_2}{\partial \bar{y}} - \bar{K}_2 \frac{\partial^2 \bar{T}_2}{\partial \bar{y}^2} - \bar{Q} = \bar{\rho}_2 c_p \frac{\partial \bar{T}_2}{\partial \bar{y}} - \bar{\rho}_2 c_p \frac{\partial \bar{T}_2}{\partial \bar{y}} = 0 \quad \dots (6)$$

$$\frac{\partial q_r}{\partial y} = 4(T - T_0) \quad i = 1, 2 \quad \dots (7)$$

Solution of the Problem

$$\theta_1 = C_1 e^{Ay} + C_2 e^{By} \quad \dots (8)$$

$$u_1 = C_3 e^{Ey} + C_4 e^{-Ey} - \frac{Re_1 P_1}{E^2} - \frac{Gr_1 C_1 e^{Ay}}{C} - \frac{Gr_1 C_2 e^{By}}{D} \quad \dots (9)$$

$$\theta_2 = C_5 e^{Fy} + C_6 e^{Gy} \quad \dots (10)$$

$$u_2 = C_7 e^{Ly} + C_8 e^{Ky} + C_9 e^{-Ly} - Re_2 P_2 K_2 - \frac{Gr_2 C_5 e^{Fy}}{H} - \frac{Gr_2 C_6 e^{Gy}}{I} \quad \dots (11)$$

Figure 2 illustrates the variation of velocity profiles across the transverse coordinate $y \in [-1.1]$ for different values of viscoelastic parameters K_1 and K_2 . Conversely, when K_2 is increased while K_1 is kept low (red curve), the velocity decreases slightly, indicating that enhanced elasticity in the second region tends to suppress the flow. Figure 3 presents velocity profiles across the channel width $\in [-1.1]$ for three different combinations of Prandtl numbers (Pr_1 and Pr_2). The Prandtl number governs the relative thickness of the momentum and thermal boundary layers. The three cases are: Black line ($Pr_1 = 1.0$ and $Pr_2 = 3.0$), Blue line $Pr_1 = 1.5$ and $Pr_2 = 1.5$ and Red line ($Pr_1 = 3.0$ and $Pr_2 = 1.0$). All profiles exhibit a smooth parabolic shape, symmetric about $y = 0$, with velocity peaking near the centre and vanishing at the channel walls. The effect of Prandtl number and Reynolds number are observed in figure 7 and figure 8 respectively. The solid line represents the temperature distribution for Region 1, and the dashed line represents the temperature distribution for Region 2. Figure 7 shows that on increasing the Prandtl number from 1.0 to 3.0 the temperature of the viscous fluid increased, but on the other hand if the Prandtl number is reduced for the viscoelastic fluid from 3.0 to 1.0, it is seen that the temperature is reduced from 3.0 to 1.5, but in the case of $Pr_1 = 1.0$ and $Pr_2 = 3.0$, i.e for a higher Prandtl number for viscoelastic fluid and a lower Prandtl number for viscous fluid, the temperature abruptly changes at $y = 0$ and increases for viscoelastic fluid.

At $y = 0$, the temperature is observed between 0.4 and 0.6. Figure 8 represents the temperature distribution with respect to the variation of Reynolds number. The figure illustrates the temperature distribution along the y-axis for different combinations of Reynolds numbers

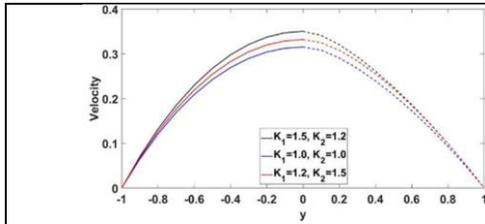


Figure 1: The effect of permeability parameters K_1 and K_2 on velocity profile. Here, $K = 0.10$, $Re_1 = 1.2$, $Re_2 = 1.2$, $\lambda = 0.8$, $\alpha_1 = 1$, $\alpha_2 = 1$, $F_1 = 0.5$, $F_2 = 0.5$, $P_1 = -0.7$, $P_2 = -0.7$, $Gr_1 = 0.1$, $Gr_2 = 0.1$, $k = 0.5$, $Pr_1 = 1.2$, $Pr_2 = 1.2$.

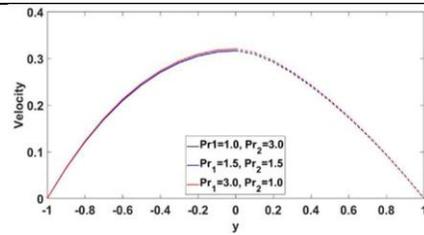


Figure 2: The effect of Prandtl number Pr_1 and Pr_2 on velocity profile. Here, $K = 0.10$, $Re_1 = 1.2$, $Re_2 = 1.2$, $\lambda = 0.8$, $\alpha_1 = 1$, $\alpha_2 = 1$, $F_1 = 0.5$, $F_2 = 0.5$, $P_1 = -0.7$, $P_2 = -0.7$, $Gr_1 = 0.1$, $Gr_2 = 0.1$, $k = 0.5$, $K_1 = 1.0$, $K_2 = 1.0$.

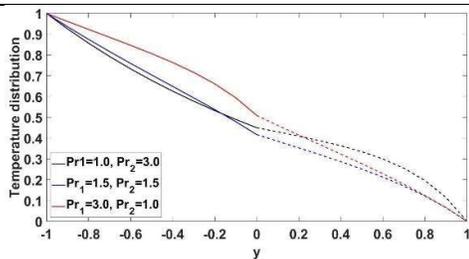


Figure 7: The effect of Prandtl number Pr_1 and Pr_2 on temperature distribution. Here, $K = 0.10$, $K_1 = 1.0$, $K_2 = 1.0$, $\lambda = 0.8$, $\alpha_1 = 1$, $\alpha_2 = 1$, $F_1 = 0.5$, $F_2 = 0.5$, $P_1 = -0.7$, $P_2 = -0.7$, $Re_1 = 1.2$, $Re_2 = 1.2$, $k = 0.5$, $Gr_1 = 0.1$, $Gr_2 = 0.1$.

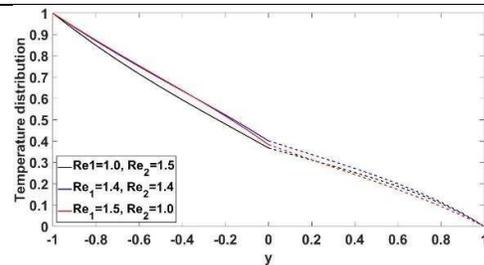


Figure 8: The effect of Reynolds number Re_1 and Re_2 on temperature distribution. Here, $K = 0.10$, $K_1 = 1.0$, $K_2 = 1.0$, $\lambda = 0.8$, $\alpha_1 = 1$, $\alpha_2 = 1$, $F_1 = 0.5$, $F_2 = 0.5$, $P_1 = -0.7$, $P_2 = -0.7$, $k = 0.5$, $Pr_1 = 1.2$, $Pr_2 = 1.2$, $Gr_1 = 0.1$, $Gr_2 = 0.1$.

Re_1 and Re_1 . The plot shows both solid and dashed lines, indicating two regions where the viscous and viscoelastic fluids flow.

Conclusion

In the present work, a heat- and mass-transfer study of multiphase flow of viscous and viscoelastic fluids is carried out between two plates. The analysis comprehensively illustrates how various physical parameters, permeability, Prandtl number, and Reynolds number, influence the temperature and velocity distributions in a two-region (viscous and viscoelastic) flow system. Overall, the study reveals that even minor asymmetries in parameter values between regions significantly affect flow and heat transfer characteristics, underscoring the importance of region-specific tuning for optimal control of such systems.

Conflict of Interest

The authors declare no conflict of interest.

References

1. Devakar, M., & Raje, A. (2019). A magnetohydrodynamic time dependent model of immiscible newtonian and micropolar fluids through a porous channel: a numerical approach. *Journal of Applied Fluid Mechanics*, 12(2), 603-615.
2. Guo, Z., Yu, F., Lin, P., Wise, S., & Lowengrub, J. (2021). A diffuse domain method for two- phase flows with large density ratio in complex geometries. *Journal of Fluid Mechanics*, 907, A38.
3. Kumar, A., & Kumar Yadav, P. (2023). Heat and mass transfer in peristaltic flow of MHD non- miscible micropolar and Newtonian fluid through a porous saturated asymmetric channel. *Waves in Random and Complex Media*, 1-45.
4. Yadav, P. K., Jaiswal, S., & Sharma, B. D. (2018). Mathematical model of micropolar fluid in two-phase immiscible fluid flow through porous channel. *Applied Mathematics and Mechanics*, 39(7), 993-1006.